

## CARPOOLING SCHEME SELECTION FOR TAXI CARPOOLING PASSENGERS: A MULTI-OBJECTIVE MODEL AND OPTIMISATION ALGORITHM

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**Abstract:** *Carpooling has been long deemed a promising approach to better utilizing existing transportation infrastructure, the carpooling system can alleviate the problems of traffic congestion and environmental pollution effectively in big cities. However, algorithmic and technical barriers inhibit the development of taxi carpooling, and it is still not the preferred mode of commute. In order to improve carpooling efficiency in urban, a taxi carpooling scheme based on multi-objective model and optimisation algorithm is presented. In this paper, urban traffic road network nodes were constructed from the perspective of passenger carpooling. A multi-objective taxi carpooling scheme selection model was built based on an analysis of the main influences of carpooling schemes on passengers. This model aimed to minimise get-on-and-get-off distance, carpooling waiting time and arriving at the destination. Furthermore, a two-phase algorithm was used to solve this model. A rapid searching algorithm for feasible routes was established, and the weight vector was assigned by introducing information entropy to obtain satisfying routes. The algorithm is applied to the urban road, the Simulation experimental result indicates that the optimisation method presented in this study is effective in taxi carpooling passengers.*

**Key words:** *traffic engineering, taxi carpooling, multi-objective optimisation, information entropy.*

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### 1. Introduction

Taxis play an important role in urban public transport systems. They provide a convenient travel service for the public that features privacy, comfort and point-to-point service; they are effective means to improve the service level and travel efficiency of urban public transport. They also expand employment opportunities (Yuan & Wu, 2014; Zhang et al., 2014; Bonarrigo et al., 2014). However, with the acceleration of urbanisation in China and the growing tension between supply and demand in the taxi market, concerned management cannot simply satisfy passenger demand by increasing the number of taxis, which will only intensify road congestion and environmental pollution (Frignal et al., 2014; Shaheen et al., 2016; Delhomme & Gheorghiu, 2016). Therefore, improving the operational efficiency and carrying rate of taxis are effective means to ameliorate taxi service level based on existing conditions. Carpooling has been adopted by several large cities as a solution. It is a voluntary behaviour in which passengers agree to take the same taxi via consultation during rush hours and the expense is shared by the carpooling passengers. Large cities, such as Beijing and

Shanghai, have implemented a taxi carpooling policy. However, most carpooling arrangements require prior appointments, and passengers should pay for reservation. Therefore, improved carpooling methods that increase efficiency and satisfy demand are the keys to the success of taxi carpooling.

### 2. Related Works

Garling et al. (2000) proposed that carpooling be applied to public transport as a solution for traffic congestion and to improve operating efficiency. They also proposed several policies to support carpooling. Luk et al. (2013) analysed the characteristics of carpoolers using a graph theory algorithm and established carpooling vehicles according to the dynamic variation of carpooling demand. Wen et al. (2014) proposed a model for matching carpooling routes based on the Global Positioning System to provide services to many-to-many carpoolers, and thus, minimise carpooling mileage and cost. Calvo et al. (2004) proposed a carpooling model based on an intelligent system to achieve an optimal carpooling allocation scheme using information and communications technology. Yan et al. (2011) achieved many-to-many matching

of carpooling using a time–space network flow technology. Baldacci et al. (2004) proposed conducting research on carpooling matching using a heuristic algorithm based on integer programming. Shao et al. (2013) proposed the concept of a matching degree based on the service requirement and characteristics of vehicles. They applied the service demand to a specific vehicle based on a heuristic algorithm. Wang et al. (2011) proposed a vehicle-sharing model based on a quantisation algorithm to achieve carpooling. Cheng et al. (2013) presented a dynamic taxi carpooling model based on a genetic algorithm. Other related studies have focused on matching carpooling routes using a heuristic algorithm or an intelligent algorithm (Waerden et al., 2015; Malodia & Singla, 2016; Huang et al., 2016; Li et al., 2014). The present study aims to establish a selection optimisation model for a taxi carpooling scheme using multi-objective decision from the perspective of passengers and to provide a basis for carpoolers to choose an optimum scheme.

### 3. Model for choosing a taxi carpooling scheme

#### 3.1. Cyberisation of urban roads

We describe road interchanges, such as crossroads, T-intersections and forks, as “Internet” nodes. We then grade the nodes of a road transport network by following the “from left to right, top to bottom” principle. Subsequently, we establish the urban road transport network  $G$  and set  $G = (v, I)$ , where  $v$  represents the Internet nodes, i.e.  $v = \{v_1, v_2, v_3, \dots, v_i\}$ . In addition,  $v_i = (x_{ix}, x_{iy})$ , where  $x_{ix}, x_{iy}$  represent the position of node  $v_i$  in the coordinate system,  $I = \{I_1, I_2, I_3, \dots, I_i\}$  and  $I_i$  represents the distance between nodes. Finally, we establish the distance matrices of the nodes as follows:

$$D = \begin{cases} d_{ij} = M, \text{ the distance between nodes } i \text{ and } j & \text{if } i \text{ and } j \text{ are adjacent nodes} \\ d_{ij} = 0, \text{ the distance between nodes } i \text{ and } j & \text{if } i = j \\ d_{ij} = \infty, \text{ the distance between nodes } i \text{ and } j & \text{if } i \text{ and } j \text{ are not adjacent nodes} \end{cases} \quad (1)$$

$D$  represents the distance of the set of nodes, and  $d_{ij}$  represents the distance between nodes  $i$  and  $j$ .

#### 3.2 Mathematical description of a taxi operation

We define  $T$  as the information set of taxis in an urban transport network,  $T = \{t_i, i \in n\}$ , where  $t_i$  denotes the information of the  $i$ th taxi. Define  $t_i = \{(t_{isx}, t_{isy}), t_{it}, t_{ip}, t_{in}, (t_{ifx}, t_{ify})\}$ , where  $(t_{isx}, t_{isy})$  denotes the position where a taxi carries its passengers,  $t_{it}$  denotes the moment when the taxi carries its passengers,  $t_{ip}$  denotes the number of passengers,  $t_{in}$  denotes the set of nodes on the route and  $(t_{ifx}, t_{ify})$  denotes the position of taxi destination.

#### 3.3 Mathematical description of a carpooler

We define  $P$  as the information set of carpoolers in the urban transport network,  $P = \{p_i, i \in n\}$ , where  $p_i$  denotes the  $i$ th carpooler group. Define  $p_i = \{(p_{isx}, p_{isy}), p_{it}, p_{ip}, p_{in}, (p_{ifx}, p_{ify})\}$ , where  $(p_{isx}, p_{isy})$  denotes the starting position of the carpooling route,  $p_{it}$  denotes the moment when the carpoolers depart,  $p_{ip}$  denotes the number of carpoolers,  $p_{in}$  denotes the set of nodes on the route during carpooling and  $(p_{ifx}, p_{ify})$  denotes the position of the destination of the carpoolers.

#### 3.4 Factors that affect the choice of a carpooling scheme

Passengers who opts for taxi carpooling are influenced by several factors when choosing a carpooling scheme, including mileage, waiting time and the optimal carpooling path.

##### (1) The distance between passengers on and off

The distance between the positions where carpoolers get on and get off, relies on the starting and final positions of carpooling. Mileage is illustrated in Fig. 1.

As shown in Fig. 1, the starting and final positions of a taxi are  $t_i(t_{isx}, t_{isy})$  and  $t_i(t_{ifx}, t_{ify})$ , the operating route is  $t_{in} = \{t_i \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6 \rightarrow t_i'\}$ , the starting and final positions of the carpoolers are  $(p_{isx}, p_{isy})$  and  $(p_{ifx}, p_{ify})$  and information on the starting and final positions is  $P_1(p_{1x}, p_{1y})$  and  $P_2(p_{2x}, p_{2y})$ . The key to calculating the positions of carpooling points  $p_1$  and  $p_2$ . The details are presented as follows.

From Fig. 1, information on the starting and final positions is confirmed. We then determine the distances, including the distances between the starting position and the nodes of the operating route and those between the final position and the nodes of the operating route.

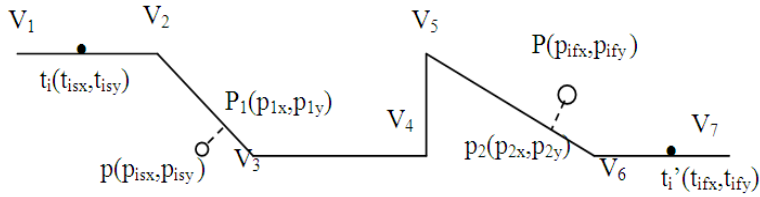


Fig. 1. The mileage of the carpooling

Furthermore, we utilise the distance calculation principle in geometry to confirm the node with the shortest operating route as follows:

$$d_{spj} = \sqrt{(p_{isx} - x_{jx})^2 + (p_{isy} - x_{jy})^2}, j \in t_{in}; \tag{2}$$

$$pn = j, j = \min(d_{spj}), j \in t_{in}; \tag{3}$$

$$d_{fpk} = \sqrt{(p_{ifx} - x_{kx})^2 + (p_{ify} - x_{ky})^2}, k \in t_{in}; \tag{4}$$

$$pm = k, k = \min(d_{fpk}), k \in t_{in}. \tag{5}$$

$d_{spj}$  represents the distances between the nodes of the operating route and the starting position of carpooling, and  $d_{fpk}$  represents the distances between the nodes of the operating route and the final position of carpooling. In addition,  $pn$  represents the node from which the distance with the starting position is the shortest, whereas  $pm$  represents the node from which the distance with the final position is the shortest.

We judge the distances of the optimal node with the starting and final positions using the distance formula. The details are as follows:

$$r_1 = \frac{\vec{v_j P} \cdot \vec{v_j v_{j-1}}}{|\vec{v_j v_{j-1}}|^2}, \quad r_2 = \frac{\vec{v_j P} \cdot \vec{v_j v_{j+1}}}{|\vec{v_j v_{j+1}}|^2}; \tag{6}$$

$$r_3 = \frac{\vec{v_k P} \cdot \vec{v_k v_{k-1}}}{|\vec{v_k v_{k-1}}|^2}, \quad r_4 = \frac{\vec{v_k P} \cdot \vec{v_k v_{k+1}}}{|\vec{v_k v_{k+1}}|^2}. \tag{7}$$

$r_1$  represents the position of the vertical line of point  $p$  on line  $v_j v_{j-1}$ ,  $r_2$  represents the position of the vertical line of point  $p$  on line  $v_j v_{j+1}$ ,  $r_3$  represents the position of the vertical line of point  $p'$  on line  $v_k v_{k-1}$  and  $r_4$  represents the position of the vertical line of point  $p'$  on line  $v_k v_{k+1}$ .

Condition 1: If  $0 < r_1 < 1$  and  $0 < r_2 < 1$ , then point  $p$  is on lines  $v_j v_{j+1}$  and  $v_j v_{j-1}$ . The distances between point  $p$  and the two lines are calculated, and the

shortest distance is identified. The carpooling point  $p_1 (p_{1x}, p_{1y})$  of point  $p$  is determined.

Condition 2: If  $r_1 \geq 1$  or  $r_1 \leq 0$ , then  $0 < r_2 < 1$ , which shows that point  $p$  is on line  $v_j v_{j+1}$ . The carpooling point  $p_1 (p_{1x}, p_{1y})$  of point  $p$  is determined.

Condition 3: If  $r_2 \geq 1$  or  $r_2 \leq 0$ , then  $0 < r_1 < 1$ , which shows that point  $p$  is on line  $v_j v_{j-1}$ . The carpooling point  $p_1 (p_{1x}, p_{1y})$  of point  $p$  is determined.

Condition 4: If  $0 < r_3 < 1$  and  $0 < r_4 < 1$ , then point  $p$  is on lines  $v_j v_{j+1}$  and  $v_j v_{j-1}$ . The distances between point  $p$  and the two lines are calculated, and the shortest distance is identified. The carpooling point  $p_2 (p_{2x}, p_{2y})$  of point  $p$  is determined.

Condition 5: If  $r_3 \geq 1$  or  $r_3 \leq 0$ , then  $0 < r_4 < 1$ , which shows that point  $p$  is on line  $v_k v_{k+1}$ . The carpooling point  $p_2 (p_{2x}, p_{2y})$  of point  $p$  is determined.

Condition 6: If  $r_4 \geq 1$  or  $r_4 \leq 0$ , then  $0 < r_3 < 1$ , which shows that point  $p$  is on line  $v_k v_{k-1}$ . The carpooling point  $p_2 (p_{2x}, p_{2y})$  of point  $p$  is determined.

The distance between the starting point of the carpoolers and that of carpooling is

$$d_s = \sqrt{(p_{isx} - p_{1x})^2 + (p_{isy} - p_{1y})^2} \tag{8}$$

The distance between the finishing point of the carpoolers and that of carpooling is

$$d_f = \sqrt{(p_{ifx} - p_{2x})^2 + (p_{ify} - p_{2y})^2} \tag{9}$$

(2) Carpooling waiting time

The waiting time of carpooling is the period from the moment a taxi arrives at the carpooling position to the moment the carpoolers arrive at the carpooling position. When the time is short, carpooling effect is good.

From Fig. 1, the waiting time of carpooling for carpoolers  $p_i$  is the time difference between the taxi from  $t_i$  to  $p_1$  and the carpoolers from  $t_i$  to  $p_1$ .

Define

$$z_2 = tc - tp, \tag{10}$$

$$tc = \frac{\sum_{i \neq j \wedge i, j \in p_m} dt_{ij} + d_{vp}}{v_t} + \Delta\tau_1 \tag{11}$$

$$tp = \frac{d_{pp1}}{v_p} + \Delta\tau_2. \tag{12}$$

$z_2$  represents the waiting time of carpooling,  $tc$  represents the time the taxi arrives at the carpooling point,  $p_{in}$  is the node matrices of the taxi from the starting point to the carpooling point,  $d_{ij}$  is the distance of each neighbouring node before the taxi arrives at the carpooling point,  $d_{vp}$  represents the distance between the carpooling point and the neighbouring node,  $\Delta\tau_1$  represents the time cost before the taxi arrives at the carpooling point caused by indefinite factors such as traffic light and traffic flow and  $v_t$  represents the average speed of the taxi.  $tp$  represents the time the carpoolers arrive at the carpooling point,  $d_{pp1}$  represents the distance the carpoolers travel to arrive at the carpooling point,  $v_p$  represents the speed of the carpoolers and  $\Delta\tau_2$  represents the time cost caused by indefinite factors such as walking speed and road calculation error.

(3) *Carpooling mileage*

Carpooling mileage is the distance between the carpooling point and the destination. The shorter the distance, the less the costs of time and expenses, the more likely the carpool will be chosen. Conversely, the longer the distance, the higher the costs of time and expenses, the less likely the carpool will be chosen.

The operating route of the taxi is set to  $t_i = (v_1, v_2, \dots, v_n)$ . The mileage of carpooling  $p_{in} = \{p_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow p_2\}$  is set. The mileage of carpooling  $z_4$  is:

$$z_4 = \sum_{i \neq j \wedge i, j \in p_m} \sqrt{(x_{ix} - x_{jx})^2 + (x_{iy} - x_{jy})^2}. \tag{13}$$

(4) *Construction selection model for taxi carpooling scheme*

Assume that the operating information of  $n$  number of taxis is available, i.e.  $T = \{t_1, t_2, t_3, \dots, t_n\}$ . Assume that the  $i$ th carpooler of an  $m$  number of carpooler groups is  $P_i$ . The multi-objective model for the  $i$ th carpooler  $P_i$  is

$$(U) = \begin{cases} \min z_1 = d_s \\ \min z_2 = d_f \\ \min z_3 = \frac{\sum_{i \neq j \wedge i, j \in p_m} dt_{ij} + d_{vp}}{v_t} + \Delta\tau_1 - \frac{d_{pp1}}{v_p} + \Delta\tau_2 \\ \min z_4 = \sum_{i \neq j \wedge i, j \in p_m} (x_{ix} - x_{jx})^2 + (x_{iy} - x_{jy})^2 \end{cases} \tag{14}$$

$$st. \begin{cases} \sqrt{(p_{ix} - p_{1x})^2 + (p_{iy} - p_{1y})^2} \leq \Delta d_1 \\ \sqrt{(p_{ix} - p_{2x})^2 + (p_{iy} - p_{2y})^2} \leq \Delta d_2 \\ \frac{\sum_{i \neq j \wedge i, j \in p_m} dt_{ij} + d_{vp}}{v_t} + \Delta\tau_1 \geq \frac{d_{pp1}}{v_p} + \Delta\tau_2 \end{cases}. \tag{15}$$

The object function of Model (14) lists the shortest carpooling distance, shortest carpooling waiting time and the optimum carpooling route (Bruglieri et al., 2014; Xu, 2016; Fahnenschreiber, 2016; Majka, 2014). Equation (15) presents the conditions of the object function, i.e. the shortest distance between the initial position of the carpoolers and the node of the operating route of the taxi should be within the range  $\Delta d_1$ . If the distance is longer than  $\Delta d_1$ , then the distance between the initial position and the carpooling point is too long; therefore, this route is unsuitable for carpooling. The shortest distance between the destination of the carpoolers and the node of the operating route of the taxi should be within the range  $\Delta d_2$ . If the distance is longer than  $\Delta d_2$ , then the distance to the destination is too long; therefore, this route is unsuitable for carpooling. The time cost of the taxi reaching the carpooling point should be higher than that of the carpoolers.

4. **Solution for the model**

Mutual contradictions exist amongst functions for an object function with the multi-objective optimisation problem. Directly determining the optimum solution using a traditional algorithm is difficult. In this study, the process of solving the multi-objective model is divided into two stages. The first stage involves solving the feasible route, whereas the second stage confirms the target weight through the information entropy, and then

comprehensively calculates the feasible routes to finally identify the optimum scheme.

*A. Searching for feasible routes*

The taxi possesses route information; hence, a route matching calculation with a single carpooler can be performed using the route information of the taxi to determine feasible routes.

**Step 1:** Calculate the distance between the taxi and the starting points of the carpoolers. If the distance is shorter than  $\Delta d$ , then perform the next calculation; otherwise, exit.

**Step 2:** Calculate the distance between the taxi and the destination of the carpoolers. If the distance is shorter than  $\Delta d$ , then perform the next calculation; otherwise, exit.

**Step 3:** Calculate the time from the taxi arriving at the carpooling point to the carpoolers arriving at the carpooling points. If the time is positive, then perform the next calculation; otherwise, exit.

**Step 4:** Calculate the matching degree. If the degree is high, then the route is feasible; otherwise, exit.

*B. Comprehensive calculation of feasible routes*

The calculation for choosing feasible routes is substantially a multi-objective decision-making problem. Information entropy (Zhang et al., 1995; Sugihakim & Alatas, 2016; Kumar & Peeta, 2015; Wolfenburg, 2014) is a common means to confirm multi-objective weights. This method is based on difference driving theory, recognises the prominence of local difference, determines the optimum weights based on the sample data, reflects the utility value of information entropy and avoids the human factor. Consequently, the index will become increasingly objective.

Assume that the number of feasible routes is  $m$ ,  $n$  is the evaluation index terms and the matrix of the index data is  $X = (x_{ij})_{m \times n}$ . The comprehensive objectives of the routes can be achieved using information entropy. The details are as follows.

**Step 1:** Establish the decision-making matrix  $X$ :

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ & & \cdots & \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{16}$$

**Step 2:** Normalise the decision-making matrix  $X$ :

$$r_{ij} = \frac{x_{ij}}{\max(x_{ij})} \tag{17}$$

Then, calculate the normalised matrix  $R$  as follows:

$$r = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ & & \cdots & \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \tag{18}$$

**Step 3:** Calculate the  $j$ th evaluation ratio  $p_{ij}$  under the  $i$ th factor as follows:

$$p_{ij} = \frac{r_{ij}}{\sum_{j=1}^n r_{ij}} \tag{19}$$

**Step 4:** Calculate entropy  $e_i$  of the  $i$ th factor:

$$e_i = -h \sum_{j=1}^m p_{ij} \ln p_{ij}, \quad h = \frac{1}{\ln m} \tag{20}$$

**Step 5:** Calculate the difference coefficient  $g_i$  of the  $i$ th factor.

The more  $e_i$  is given, the smaller the difference of the evaluation and the less the effect of the factor in the comprehensive evaluation. We define the difference coefficient  $g_i = 1 - e_i$ . The higher the value of  $g_i$ , the more important the factor will be.

**Step 6:** Define weight  $w_i = \frac{g_i}{\sum_{i=1}^m g_i}$ , where  $w_i$  is the

weight confirmed using the entropy weight method.

**Step 7:** Perform the comprehensive calculation of the feasible routes.

$$k_i = \sum_{j=1}^n w_i \cdot r_{ij}, \quad i = 1, 2, \dots, n \tag{21}$$

Arrange the feasible routes from large to small based on the comprehensive objectives; the former routes are the optimal routes.

**5. Simulation results and performance evaluation**

Fig. 2 presents the urban traffic transformation Figure G of several counties in Lanzhou, with 37 nodes. Their coordinates are shown in Table 1, and the distances between each node are listed in Matrix (26). Assume that each road in Figure G is bidirectional. The route information of the taxis is randomly generated based on the attributes of the taxis and passengers. The route information is shown in Table 2, the travel information of the passenger is  $P_i$ , the starting position is (3345, 2040), the departure time is 9:50 am, the number of carpoolers is one and the destination is (6675,6435). The multi-objective optimal model for choosing a carpooling point is established based on the coordinates of the nodes, the distance matrix of the nodes, the route information of the taxis, the positions where passengers get on and get off, carpooling waiting time and the similarity of routes. The weight of the decision-making matrix of the feasible routes is then confirmed using information entropy theory, and the comprehensive objectives of the feasible routes are determined as shown in Table 3.



Fig. 2. The nodes of the road network

$$D = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \dots & 35 & 36 & 37 \\ 1 & 0 & 0.74 & \infty & \infty & \infty & \dots & \infty & \infty & \infty \\ 2 & 0.74 & 0 & 0.49 & \infty & 0.82 & \dots & \infty & \infty & \infty \\ 3 & \infty & 0.49 & 0 & 0.85 & \infty & \dots & \infty & \infty & \infty \\ & & & \dots & & & & & & \\ 35 & \infty & \infty & \infty & \infty & \infty & \dots & 1.14 & \infty & \infty \\ 36 & \infty & \infty & \infty & \infty & \infty & \dots & 1.01 & 0 & 5.87 \\ 37 & \infty & \infty & \infty & \infty & \infty & \dots & \infty & 1.50 & 0 \end{bmatrix} \quad (26)$$

Table 1. The coordinates of the nodes

No	Coordinate $(x_{ix},x_{iy})$	No	Coordinate $(x_{ix},x_{iy})$	No	Coordinate $(x_{ix},x_{iy})$	No	Coordinate $(x_{ix},x_{iy})$
1	300 1665	10	2565 3645	19	4710 5415	28	6075 5775
2	825 1140	11	3840 2280	20	5475 795	29	5910 870
3	1155 780	12	3495 4425	21	5355 1830	30	6345 1875
4	1440 465	13	3720 4620	22	5355 2460	31	6855 945
5	1545 1530	14	4005 4215	23	5355 3600	32	6870 2025
6	1410 2655	15	4695 765	24	5235 5925	33	6900 2655
7	3150 645	16	4710 1710	25	5595 1890	34	6885 3660
8	3045 1440	17	4695 2385	26	5610 2505	35	6885 4800
9	2850 2175	18	4725 3615	27	5865 4725	36	6900 5805
						37	6870 7305

Table 2. The routes information of the taxis

No	Initial position	Time	Operating routes	Destination
1	(1830,480)	9:30	7→8→9→11→17→22→26→33	(6885,7410)
2	(2445,1965)	9:55	9→11→17→22→26→33→34→35→36	(6870,6900)
3	(2850,2160)	10:00	9→11→17→18→19→24→37	(6870,5310)
4	(495,1875)	9:20	1→6→10→12→13→19→24→37→36	(6870,6660)
5	(2070,3225)	9:58	10→9→11→17→22→26→33→34→35→36	(6840,1575)
6	(3060,1140)	9:54	8→9→11→12→13→19→24→37	(6870,6660)
7	(2275,2655)	10:01	9→11→17→18→23→34→35→36	(6900,6290)
8	(6870,1170)	9:20	31→32→33→34→35→36	(6900,6105)
9	(420,1485)	9:10	1→2→3→4→7→15→20→29	(6285,840)
10	(825,2235)	9:15	1→6→10→12→13→19→24	(5670,6360)

Table 3. The carpooling schemes and their comprehensive objectives

The routes information of the taxis	Objectives	Z <sub>1</sub> (m)	Z <sub>2</sub> (m)	Z <sub>3</sub> (min)	Z <sub>4</sub> (m)
9→11→17→22→26→33→34→35→36	0.9799	73	50	3.95	1360
9→11→17→18→23→34→35→36	0.8752	73	80	9.15	1282
10→9→11→17→22→26→33→34→35→36	0.8467	73	109	7.62	1354
9→11→17→18→19→24→37	0.5415	120	352	9.35	1940
8→9→11→12→13→19→24→37	0.4461	163	159	5.23	3423
7→8→9→11→17→22→26→33	Infeasible routes				
1→6→10→12→13→19→24→37→36	Infeasible routes				
31→32→33→34→35→36	Infeasible routes				
1→2→3→4→7→15→20→29	Infeasible routes				
1→6→10→12→13→19→24	Infeasible routes				

Five feasible and five infeasible routes are identified from Table 3. We compare the comprehensive objectives of the five feasible routes, and then determine that the values of Z<sub>1</sub> and Z<sub>3</sub> significantly affect carpooling. Their weights are 0.3859 and 0.3506, respectively, which reflect different driving traits based on information entropy. Carpooler P<sub>1</sub> can choose the optimum carpooling scheme based on the comprehensive objectives.

## 6. Conclusion

(1) We establish the objective function to identify the shortest distance of carpoolers getting on and getting off, the shortest carpooling waiting time and the shortest distance of arriving at the destination via carpooling. We also establish the optimum model for carpooling that considers a single carpooling time. We solve the multi-objective optimal scheme using the search algorithm and information entropy theory.

(2) We calculate the comprehensive objectives of the feasible routes. The proposed model can identify the optimum carpooling scheme quickly and effectively.

(3) This study focuses on optimising taxi carpooling based on assumed conditions. Factors, such as road congestion, and the determination of operating routes are not considered. Rigorously describing unknown routes from the perspective of optimisation and further research on the optimisation of multi-time carpooling are areas for further discussion in the follow-up stage.

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